Correlation decay in quantum chaotic billiards with bulk or surface disorder

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We study the decay properties of correlation functions in quantum billiards with surface or bulk disorder. The quantum system is modeled by means of a tight-binding Hamiltonian with diagonal disorder, solved on $L\times L$ clusters of the square lattice. The correlation function is calculated by launching the system at $t=0$ into a wave function of the regular (clean) system and following its time evolution. The results show that the correlation function decays exponentially with a characteristic correlation time (inverse of the Lyapunov exponent λ). For small enough disorder the Lyapunov exponent is approximately given by the imaginary part of the self-energy induced by disorder. On the other hand, if the scaling of the Lyapunov exponent with L is investigated by keeping constant l/L, where l is the mean free path, the results show that $\lambda\propto 1/L$. [S1063-651X(99)00607-8]

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I. INTRODUCTION

The Lyapunov exponent in quantum systems whose classical analogs behave chaotically are commonly investigated by studying the stability of periodic orbits of the classical system [1]. The procedure consists of launching the quantum system at $t=0$ along a periodic orbit, and calculating the overlap between the wave function at $t=0$ and that at a finite time. The time decay of this correlation function gives the Lyapunov exponent. Although the procedure is straightforward, only few studies of the dynamical behavior of quantum chaotic systems have yet been published [1–4].

In this paper we investigate correlation functions in quantum billiards with either bulk or surface disorder. Our model consists of a simple tight-binding Hamiltonian with diagonal disorder solved on regular (square) clusters of the square lattice. Thus, while the shape of the system is regular, disorder triggers the expected complex behavior. Two types of bulk disorder are investigated, namely, the standard bulk disorder in which all sites are assumed to have atomic levels with random energies (L^2 impurities, L being the linear size of the cluster) or with only $4L$ bulk impurities [5]. The latter has in common with the surface disorder model the fact that in both cases the density of impurities decreases with the size of the system. We show that the last two models behave in a very similar way. The case of surface disorder can be also viewed as an efficient implementation of surface roughness [6–8]. The scaling of the magnitudes investigated here with the size of the system was investigated keeping constant the ratio l/L, where l is the mean free path of the system (see [9,10] and below). The results show that in all cases investigated here, and for sufficiently small values of the disorder parameter, the Lyapunov exponent is given by the imaginary part of the self-energy or, equivalently, the relaxation time, induced by disorder. Although this result is consistent with Laughlin's suggestion in the sense that chaos induces a relaxation time whose inverse approximately gives the Lyapunov exponent [2,11,12], some important differences between the two approaches are discussed below (see Sec. IV).

II. MODEL AND METHODS

A. Hamiltonian

The tight-binding Hamiltonian used in this work includes one atomic orbital per lattice site and a hopping term restricted to nearest-neighbors sites. This Hamiltonian can be written as

$$\hat{H}=\sum_i \omega_i \hat{c}_i^{\dagger}\hat{c}_i-\sum_{\langle ij\rangle} V_{ij}\hat{c}_i^{\dagger}\hat{c}_j\,, \tag{1}$$

where the operator $\hat{c}_i$ destroys an electron on site i, and V_{ij} is the hopping integral between sites i and j (the symbol $\langle ij\rangle$ denotes that the sum is restricted to nearest neighbor sites). We take $V_{ij}=V=1$ and the lattice constant as the unit of length. The energy of the atomic orbital at lattice sites i, ω_i, is randomly chosen between $-W/2$ and $W/2$ according to three different models in which the sum in the first term of Hamiltonian (1) runs over: (a) all lattice sites, (b) only over $4L$ randomly chosen bulk sites, and (c) only over the surface sites. For the remaining lattice sites we take $\omega_i=0$. Calculations have been carried out on clusters of sizes up to $L=63$. Averaging sets include around 16×10^3 energy levels. The Schwarz algorithm was used to compute the whole spectrum [13].

B. Correlation functions

The dynamical behavior of Hamiltonian (1) was investigated as follows. The wave functions of the chaotic billiard are

$$|\psi_\alpha\rangle = \sum_{i=1}^{N} a_{\alpha i}|i\rangle, \tag{2}$$

where $a_{\alpha i}$ is the amplitude of the eigenstate α on the lattice site i. At $t=0$, the disordered system is launched onto an eigenfunction of the ordered cluster (billiard) characterized by a wave vector $\mathbf{k}$

$$|\Psi(0)\rangle = |\phi_{\mathbf{k}}\rangle = \sum_{\alpha=1}^{N} b_{\alpha\mathbf{k}}|\psi_\alpha\rangle, \tag{3}$$

where the constants $b_{\alpha\mathbf{k}}$ are given by

$$b_{\alpha\mathbf{k}} = \langle\psi_\alpha|\phi_{\mathbf{k}}\rangle. \tag{4}$$

The time dependent wave function of the disordered system is

$$|\Psi(t)\rangle = \sum_{\alpha=1}^{N} b_{\alpha\mathbf{k}} e^{-it\epsilon_\alpha}|\psi_\alpha\rangle. \tag{5}$$

where ϵ_α is the energy of the eigenstate α of the disordered system. As the unit of energy is the hopping integral (V), the time is measured in units of $\hbar/V$. Finally, the correlation function is given by

$$C(t) = \langle\Psi(0)|\Psi(t)\rangle. \tag{6}$$

In carrying out the numerical calculations we have launched the system at $t=0$ always onto the same ordered eigenstate. We have taken an eigenstate with momentum $\mathbf{k} = (\pi/4, 3\pi/8)$ and chosen L in such a way that this state was always an eigenstate of the system ($L = 7, 15, 23, \ldots, 63$). This procedure reduces statistical errors in a way cheaper than averaging over a finite energy range. We have checked that the results do not significantly depend on the chosen ordered eigenstate.

C. Self-energy

We calculate the self-energy induced by disorder in an ordered state of momentum $\mathbf{k}$ and energy $\epsilon_{\mathbf{k}}$ from Dyson's equation,

$$\hat{\Sigma}(\epsilon_{\mathbf{k}}) \equiv \hat{G}_0^{-1}(\epsilon_{\mathbf{k}}) - \langle\hat{G}(\epsilon_{\mathbf{k}})\rangle^{-1}, \tag{7}$$

where $\hat{\Sigma}$ is the self-energy operator, $\hat{G}_0$ the unperturbed Green function associated to the nondiagonal term of Hamiltonian (1), and $\langle\hat{G}\rangle$ the perturbed Green function averaged over disorder realizations. Then, in order to calculate the matrix element $\Sigma(\mathbf{k},\mathbf{k};\epsilon_{\mathbf{k}})$, we only have to determine the inverse of the averaged perturbed Green function in the $\mathbf{k}$-space basis using the site representation. The details of the calculation are discussed in Refs. [9,10].

The qualitative behavior of the selfenergy can be obtained by second-order perturbation theory [14]. Averaged real-space matrix elements of the self-energy operator are approximately given by

$$\Sigma(i,j;\epsilon_{\mathbf{k}}) \approx \delta_{ij}\langle\omega_i^2\rangle G_0(i,i;\epsilon_{\mathbf{k}}), \tag{8}$$

for those sites i for which the diagonal energy ω_i fluctuates. tranforming to $\mathbf{k}$-space and using a rough approximation for the imaginary part of the diagonal elements of the unperturbed Green function ($\mathrm{Im}[G_0(i,i;\epsilon_{\mathbf{k}})] \sim \pi/(4d)$, with d being the dimension of the physical space), we obtain for surface disorder or $4L$ bulk impurities

$$\mathrm{Im}\,\Sigma(\mathbf{k},\mathbf{k};\epsilon_{\mathbf{k}}) \approx -\frac{\pi}{24L}W^2. \tag{9}$$

This indicates that, within the limits of validity of the approximations used to derive this equation, the effects of $4L$ bulk impurities and of surface disorder are very similar. Instead for bulk disorder (L^2 impurities) this equation is replaced by

$$\mathrm{Im}\,\Sigma(\mathbf{k},\mathbf{k};\epsilon_{\mathbf{k}}) \approx -\frac{\pi}{24L}\frac{W^2 L}{4}. \tag{10}$$

Equations (11) and (12) indicate that the effects of surface and bulk disorder can be made very similar if

$$W^2_{\mathrm{surface}} \approx L\,W^2_{\mathrm{bulk}}/4, \tag{11}$$

for a given linear size L of the system.

The effect of disorder can also be characterized by means of the relaxation time. Using the approximation of Eq. (8) it is easy to show that [14]

$$\tau_{\mathbf{k}} = \frac{1}{2|\mathrm{Im}\,\Sigma(\mathbf{k},\mathbf{k};\epsilon_{\mathbf{k}})|}, \tag{12}$$

or, equivalently, by the mean free path $l_{\mathbf{k}}$ which is defined as

$$l_{\mathbf{k}} \equiv v_{\mathbf{k}}\tau_{\mathbf{k}}, \tag{13}$$

where $v_{\mathbf{k}}$ is the velocity of a state with momentum $\mathbf{k}$. The mean free path is measured in units of the lattice constant a, and the relaxation time in units of the ratio $\hbar/V$.

In investigating the scaling of the magnitudes herewith studied with the size of the system L, we keep the ratio l/L constant. This is accomplished by keeping constant W for surface disorder and $4L$ bulk impurities, and the right-hand side (RHS) of Eq. (11) for bulk disorder. This procedure impedes localization in the case of bulk disorder and allows a meaningful comparison of the different models. On the other hand, this ensures that not only the models with either $4L$ bulk impurities or surface disorder [6], but also that with L^2 impurities, will show signs of chaotic behavior (eigenvalues distributed according to the Wigner-Dyson distribution).

The validity of Eqs. (9) and (10) is illustrated in Fig. 1, where the results given by those equations along with the numerical results for the imaginary part of the self-energy in the three models of disorder here considered, are shown. It is noted that for small values of the disorder parameter Eqs. (9) and (10) fit very approximately the numerical results. Instead as the strength of disorder increases, the value of the self-energy for the model with $4L$ impurities, saturates. This is a consequence of a gradual decoupling of the impurities from the rest of the cluster, due to the very different values of the diagonal term in Hamiltonian (1) associated to the impurity

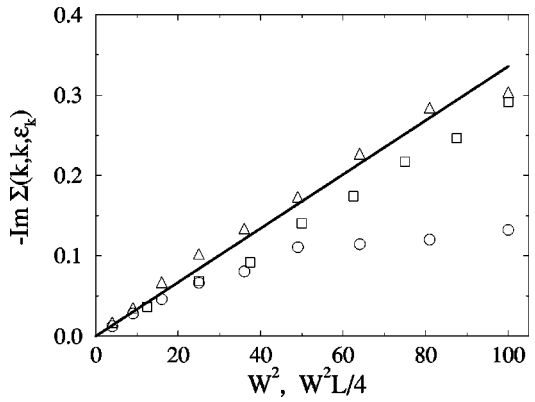

FIG. 1. Imaginary part of the self-energy induced by disorder on an eigenstate with momentum $\mathbf{k}=(\pi/4,3\pi/8)$, as a function of the disorder parameter (see text). The results correspond to 39×39 clusters and 10 realizations of disorder. The symbols correspond to surface disorder (triangles), $4L$ bulk impurities (circles), and L^2 impurities (squares). The straight line is that given by Eqs. (9) and (10). Magnitudes are dimensionless: the self-energy and W are measured in units of the hopping energy V and L in units of the lattice constant a.

sites and to the rest of the cluster sites. This decoupling does also occur in the case of surface disorder [9]. In fact for large W the surface layer decouples from the inner cluster of linear size $L-2$, leading to $l\rightarrow\infty$, or, alternatively, a self-energy tending to zero. The reason why this is not observed in Fig. 1 is the failure of perturbation theory as used here (see [9] for a thorough discussion of this point). Finally we note that the numerical results also indicate that the real part of the self-energy is more than an order of magnitude smaller than the energy of the unperturbed eigenstate, for the largest values of the disorder parameter investigated here.

III. RESULTS

As indicated by the results and fitting of Fig. 2, the correlation function $C(t)$ at short times behaves very approximately as [15–17]

$$C(t)=\exp(-[\lambda+i\epsilon_{\mathbf{k}}]t), \tag{14}$$

where λ is the Lyapunov exponent (measured in units of the hopping integral V). A fitting of the numerical results for the absolute value of $C(t)$ at sufficiently short times allows to determine λ. At longer times the behavior of $C(t)$ is substantially more complex (see Fig. 3). $C(t)$ shows the typical features of chaotic systems with regions in which it is finite alternating with others where it is very small (this point is further discussed below). We have to point out that this behavior of $C(t)$ is significantly different from that reported in Refs. [3,4], namely, Gaussian-like peaks with maxima at $2\pi n/\epsilon_{\mathbf{k}}$, $n=0,1,2,\ldots$, modulated by an exponential.

The Lyapunov exponent was obtained from fittings of the time evolution of the modulus of the correlation function, as illustrated in Fig. 4. It is noted that the time range in which the exponential behavior holds increases with the system size. The numerical results for λ as a function of L are reported in Fig. 5, whereas the fitted curves are reported in Table I. As pointed out above, the scaling of λ with L was

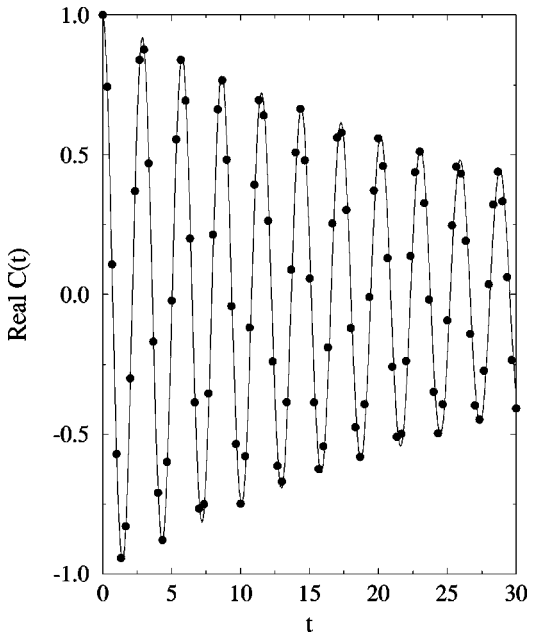

FIG. 2. Short time behavior of the real part of the correlation function (filled circles) for the wave function of the quantum system with $4L$ bulk impurities, launched at $t=0$ on a wave function of the ordered billiard with wave vector $\mathbf{k}=(\pi/4,3\pi/8)$ (see text). The results correspond to a disorder parameter $W=4$ and a linear size $L=63$. The continuous curve corresponds to $C(t)=\cos(2.18t)\exp(-0.0279t)$. The unit of time is $\hbar/V$, V being the hopping energy.

investigated keeping the ratio l/L constant [see discussion concerning Eqs. (9)–(11)]. As shown in that table, in the macroscopic limit ($L\rightarrow\infty$) the Lyapunov exponent vanishes as $1/L$. The dependence of λ on the disorder parameter is shown in Fig. 6 along with the self-energy derived from the approximate expressions of Eqs. (9) and (10). The results

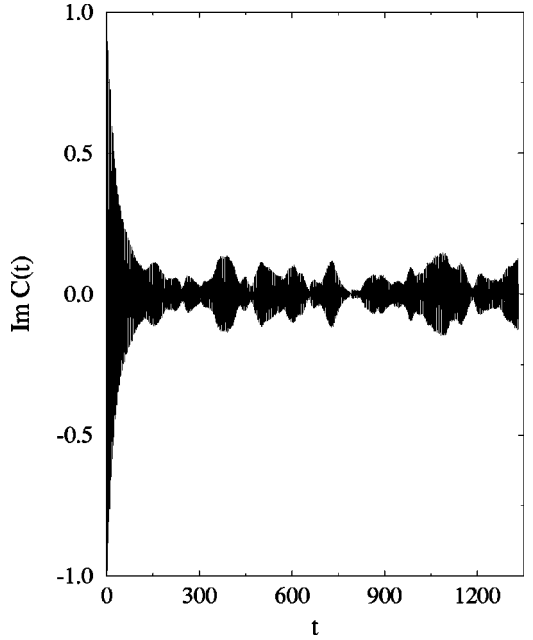

FIG. 3. Long time behavior of the imaginary part of the correlation function for the quantum system with $4L$ bulk impurities. The rest of the parameters as in Fig. 2.

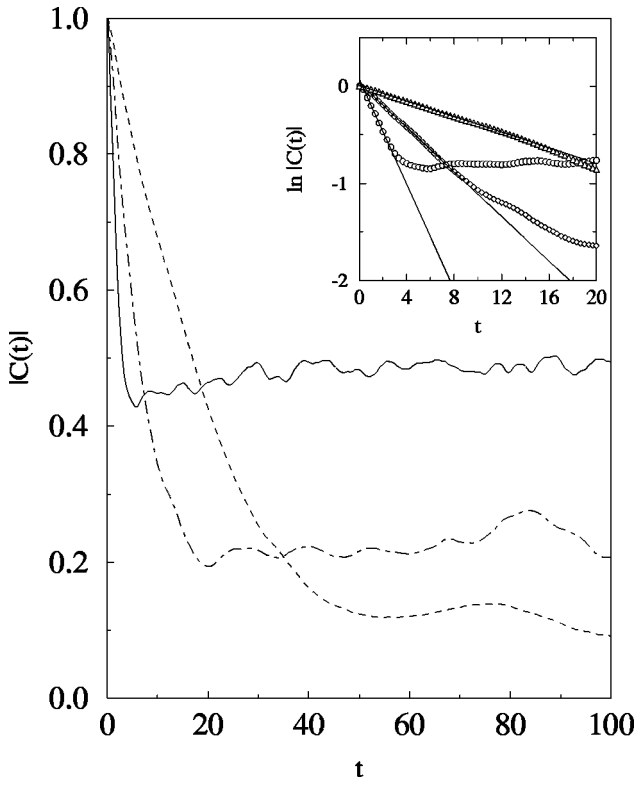

FIG. 4. Modulus of the correlation function (averaged over several realizations) versus time, for the wave function of the disordered system launched at $t=0$ on the wave function of the ordered system with wave vector $\mathbf{k}=(\pi/4,3\pi/8)$ (see text). The results correspond to a disorder parameter $W=4$ and the following system sizes: $L=7$ (broken line), $L=23$ (continuous line), and $L=63$ (dotted line). Inset: short time fittings of $C(t)$ with $\exp(-\lambda t)$, λ being the Lyapunov exponent (see text). Units as in Fig. 2.

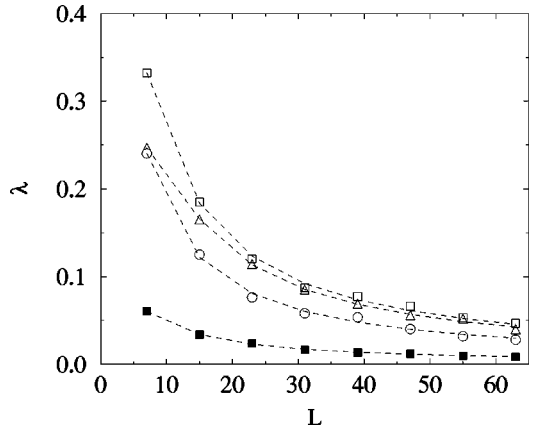

FIG. 5. Lyapunov exponent λ for surface disorder (triangles), bulk disorder with L^2 impurities (squares), and $4L$ bulk impurities (circles) as a function of the system size L. The wave number of the ordered wave function into which the system was launched at $t=0$ is $\mathbf{k}=(\pi/4,3\pi/8)$. The results correspond to the following values of the disorder parameter W: surface disorder $W^2=16$ (open triangles), $4L$ bulk impurities $W^2=16$ (open circles), and L^2 impurities, $W^2L/4=5$ (filled squares) and 27.5 (open squares). The fitted curves are given in Table I. All magnitudes are dimensionless: W and λ are measured in units of the hopping energy V and L in units of the lattice constant.

indicate that for small values of the disorder parameter the Lyapunov exponent coincides with the imaginary part of the self-energy. This is further illustrated by the fact that the two magnitudes scale as $1/L$ in the large L limit. Moreover, the results of Table I show that the constant that multiplies the term $1/L$ in the fittings of the numerical results is very similar to that of Eqs. (9) and (10). The agreement is particularly important for small values of the disorder parameter as indicated by the results for both types of bulk disorder shown in that table. At large W, the Lyapunov exponent for bulk disorder and $4L$ bulk impurities is still surprisingly very similar to the imaginary part of the self-energy. We do not have a sound explanation for this agreement (see below). This is not the case of surface disorder for which the Lyapunov exponent in the high W limit significantly differs from the imaginary part of the self-energy, being instead similar to the Lyapunov exponent for 4L impurities. The different behaviors of the two magnitudes may be a consequence of the problems found in the calculation of the self-energy discussed above.

It is interesting to note that a behavior of the correlation function like that of Eq. (14) for all times should have only been expected if the distribution of weights $b_{\alpha\mathbf{k}}$ in Eq. (3) or, alternatively, the Fourier transform of $C(t)$, would have had a Lorentzian shape, namely,

$$S(\epsilon)=\frac{\lambda}{\pi[\lambda^2+(\epsilon-\epsilon_{\mathbf{k}})^2]}. \tag{15}$$

In such a case the variance of this distribution would have completely determined the decay of $C(t)$, and, thus, coincided with the Lyapunov exponent. The long time behavior of the correlation function shown in Fig. 3 clearly suggests that this is not the case. This is further illustrated by the numerical results for the variance σ of the distribution of the weights $b_{\alpha\mathbf{k}}$ in Eq. (3) reported in Figs. 7 and 8 and in Table I. As shown in those figures neither the scaling of σ nor its dependence on the disorder parameter coincide with those of the Lyapunov exponent discussed above.

IV. DISCUSSION

The results of the previous section indicate that the Lyapunov exponent has no relation with σ and, instead, is given by the imaginary part of the self-energy for small values of the disorder parameter. In this section we first discuss an analytical derivation of σ that supports the numerical results. We also justify the relation between λ and the imaginary part of the self-energy.

The accuracy of our numerical results for σ is supported by an analytic (and approximate) derivation of the scaling of this variance with L, which closely follows the numerical procedures used to obtain the results of Figs. 7 and 8. Let us expand an eigenstate of the ordered system with wave vector $\mathbf{k}$ and energy $\epsilon_{\mathbf{k}}$, in terms of the α eigenstates of the disordered system. The average energy $\langle\epsilon\rangle$ is then given by

$$\langle\epsilon\rangle=\langle\mathbf{k}|\hat{H}|\mathbf{k}\rangle=\epsilon_{\mathbf{k}}+\langle\mathbf{k}|\hat{V}|\mathbf{k}\rangle, \tag{16}$$

TABLE I. Fittings of the numerical results for the Lyapunov exponent (λ) and the variance of the distribution of disordered eigenfunctions (σ), as a function of the system size L, for the various models of disorder investigated in this work (see text). The fitted curves are $\lambda = aL^{-1} + bL^{-3}$ and $\sigma = cL^{-1/2} + dL^{-3/2}$. The results correspond to the case in which the disordered system was launched at $t=0$ on the wave function of the ordered system with wave vector $\mathbf{k}=(\pi/4, 3\pi/8)$ (see text). The mean free path l of this wave function is also given. All magnitudes are dimensionless: W, λ, and σ are measured in units of the hopping energy V, and L in units of the lattice constant a.

Disorder	Disorder parameter	l/L	a	λ b	a [Eqs. (9) and (10)]	c	σ d	c [Eqs. (21) and (22)]
Surface	$W^2=16$	0.45	2.7	−47.4	2.09	2.64	−2.71	2.31
Bulk (L^2)	$LW^2/4=5$	1.68	0.54	−5.7	0.65	1.29	−0.24	1.29
	$LW^2/4=27.5$	0.43	2.88	−27.3	3.6	3.03	−0.57	3.03
Bulk ($4L$)	$W^2=4$	1.82	0.48	−3.42	0.52	1.16	−0.002	1.16
	$W^2=16$	0.61	1.87	−9.19	2.09	2.33	−0.62	2.31

where $\hat{V}$ is the perturbing part of the Hamiltonian operator [diagonal term in Eq. (7)]. On the other hand, the average of the energy square is

$$\langle \epsilon^2 \rangle = \epsilon_{\mathbf{k}}^2 + 2\epsilon_{\mathbf{k}} \langle \mathbf{k}|\hat{V}|\mathbf{k}\rangle + \langle \mathbf{k}|\hat{V}^2|\mathbf{k}\rangle, \tag{17}$$

and, thus, the standard deviation is

$$\sigma = \sqrt{\langle \epsilon^2 \rangle - \langle \epsilon \rangle^2} = \sqrt{\langle \mathbf{k}|\hat{V}^2|\mathbf{k}\rangle - \langle \mathbf{k}|\hat{V}|\mathbf{k}\rangle^2}. \tag{18}$$

Now we explicitly calculate the matrix elements of the perturbing potential and its square, averaged over disorder realizations. To this end we expand the Bloch wave functions of the ordered system in terms of the atomic orbitals $|i\rangle$ located on sites i as $|\mathbf{k}\rangle = (1/L)\Sigma_{i=1}^{N} c_{\mathbf{k}i}|i\rangle$. The matrix elements are then rewritten as

$$\langle \mathbf{k}|\hat{V}|\mathbf{k}\rangle = \frac{1}{L^2} \sum_i c_{\mathbf{k}i}^2 \omega_i, \tag{19a}$$

$$\langle \mathbf{k}|\hat{V}^2|\mathbf{k}\rangle = \frac{1}{L^2} \sum_i c_{\mathbf{k}i}^2 \omega_i^2, \tag{19b}$$

where ω_i are the energies of the atomic orbitals located on impurity sites i. Averaging over disorder realizations leads to

$$\langle \langle \mathbf{k}|\hat{V}|\mathbf{k}\rangle \rangle = \frac{1}{L^2} \sum_i c_{\mathbf{k}i}^2 \langle \omega_i \rangle = 0, \tag{20a}$$

$$\langle \langle \mathbf{k}|\hat{V}|\mathbf{k}\rangle^2 \rangle = \frac{1}{L^4} \sum_i c_{\mathbf{k}i}^2 \sum_j c_{\mathbf{k}j}^2 \langle \omega_i \omega_j \rangle \approx \frac{W^2}{3L^3}, \tag{20b}$$

$$\langle \langle \mathbf{k}|\hat{V}^2|\mathbf{k}\rangle \rangle = \frac{1}{L^2} \sum_i c_{\mathbf{k}i}^2 \langle \omega_i^2 \rangle \approx \frac{W^2}{3L}. \tag{20c}$$

In deriving these results we have taken into account that averaging over disorder realizations gives $\langle \omega_i \rangle = 0$, and, $\langle \omega_i \omega_j \rangle = (W^2/12)\delta_{i,j}$. Furthermore, we have taken $c_{\mathbf{k}i} \approx 1$ for all i. This neglects all dependence on $\mathbf{k}$. Then, the variance for surface disorder and $4L$ bulk impurities results to be

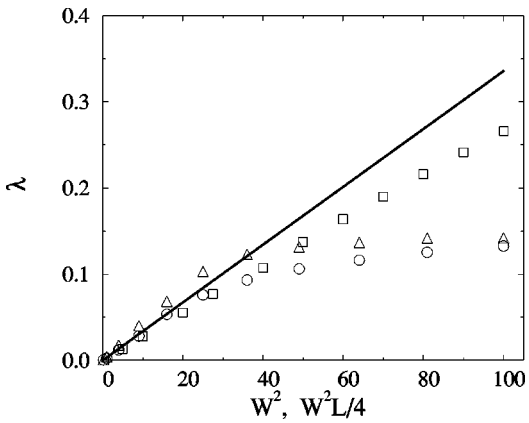

FIG. 6. Lyapunov exponent λ as a function of the relevant disorder parameter in each case (W^2 for surface disorder and $4L$ bulk impurities, and $W^2L/4$ for L^2 bulk impurities; see text). The results correspond to 39×39 clusters. The rest of the parameters, symbols, units, and procedures are those of Fig. 5. The straight line is that of Eqs. (9) and (10).

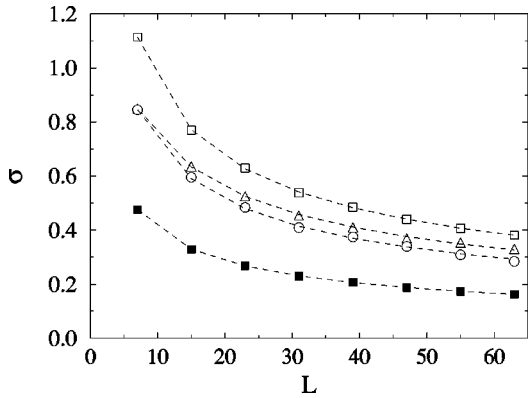

FIG. 7. Same as Fig. 5 for the half-width σ of the distribution of the weights of the wave functions of the disordered system [see Eq. (3)]. σ is measured in units of the hopping energy.

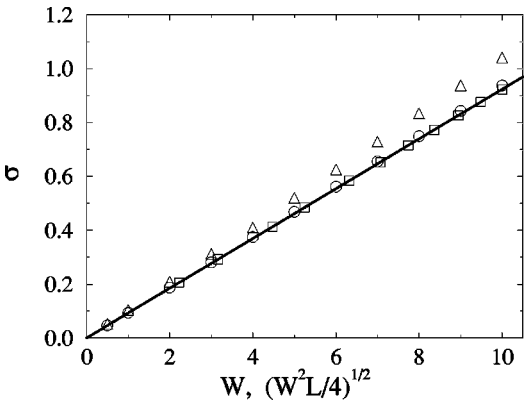

FIG. 8. Same as Fig. 6 for the half-width σ of the distribution of the weights of the wave functions of the disordered system [see Eq. (3)]. The straight line is that of Eqs. (21) and (22). Units as in Figs. 6 and 7.

$$\sigma \approx \sqrt{\frac{W^2}{3L}}, \tag{21}$$

whereas for bulk disorder the result is

$$\sigma \approx \sqrt{\frac{W^2 L/4}{3L}}. \tag{22}$$

The accuracy of Eqs. (21) and (22) is illustrated by the results reported in Table I and Fig. 8. Although in all cases the agreement with the numerical results is remarkable, it is particularly important for bulk disorder. The reason why it is not so good for surface disorder is likely a consequence of the stronger breaking of translational invariance induced by surface disorder. This makes poorer the approximation in which we neglected the dependence of the matrix elements of the potential on $\mathbf{k}$ [see discussion concerning Eqs. (20a)–(20c)]. The agreement between the approximate formulas and the numerical results support our conclusion concerning the uncorrelation between the Lyapunov exponent and σ.

The argument that justifies the relation between the imaginary part of the self-energy and the Lyapunov exponent goes as follows. The correlation function can be written in terms of the self-energy as

$$C(t) = \langle \Psi(0) | e^{-i\hat{H}t} \Psi(0) \rangle = \langle \phi_{\mathbf{k}} | e^{-i(\hat{H}_0 + \hat{\Sigma})t} | \phi_{\mathbf{k}} \rangle, \tag{23}$$

where $\hat{H}_0$ is the nondiagonal part of Hamiltonian (1). In general the operator $\hat{\Sigma}$ does not commute with $\hat{H}_0$ and the calculation of this expectation value is not straightforward. However, for small disorder the selfenergy is approximately proportional to the unperturbed Green's function [see Eq. (8)] and, therefore, the two operators in the above equation approximately commute. This leads to

$$C(t) \approx \exp[-i(\epsilon_{\mathbf{k}} + \Sigma(\mathbf{k}, \mathbf{k}; \epsilon_{\mathbf{k}}))t]. \tag{24}$$

This is the behavior of the correlation function we found numerically for small disorder. In fact, as the real part of the self-energy is much smaller than $\epsilon_{\mathbf{k}}$ (see above), Eq. (24) gives a correlation function which oscillates with a periodicity given by $\epsilon_{\mathbf{k}}^{-1}$ and is modulated by a decaying exponential as in Fig. 2. This indicates that at least for small W the Lyapunov exponent is given by the imaginary part of the self-energy, in agreement with the numerical results discussed in the preceding section. However, there are no reasons why this argument should also hold at large W and, thus, it cannot explain the similarity between the two magnitudes found numerically for both types of bulk disorder in the whole range of W explored in this work.

Note that the approximation used to derive Eq. (24) coincides with that followed to write the relaxation time in terms of the imaginary part of the self-energy (see Ref. [14]). This is in line with the expected equivalence between the relaxation time induced by disorder and the inverse of the Lyapunov exponent which characterizes the decay of the correlation function defined in Eq. (6).

It is interesting to comment on the argument put forward by Laughlin to justify the existence of a relaxation time induced by chaos. Laughlin approach considers classical particles (electrons) scattered by a crystalline array of hard spheres (atoms) of radius R with unit cell volume Ω. This system behaves chaotically in the classical limit. The rate at which the electrons collide with the spheres (relaxation time) is given by $\tau^{-1} = v(\pi R^2/\Omega) \approx v\pi/R$, where v is the electron velocity. Then, the uncertainty in the impact parameter Δx evolves with time approximately as [2,11,12]

$$\Delta x(t) = \Delta x(0) e^{t/\tau}. \tag{25}$$

This allows us to identify the Lyapunov exponent λ with the inverse of the relaxation time τ^{-1}. Transferring this argument to the quantum case has two main drawbacks. (i) It is well-known that a perfect periodic array of atoms does not produce any scattering of quantum particles [18] and, as a consequence, $\tau = \infty$. (ii) In the absence of decoherence, a positive Lyapunov exponent, as implied by Eq. (25), is not expected in quantum mechanics; correlation functions, the magnitudes commonly calculated in the quantum limit [1,3], show a decaying behavior and, thus, a Lyapunov exponent less than zero. A further difference with respect to the present analysis is the fact that our relaxation time tends to infinity in the thermodynamic limit. Ensuring that the system behaves chaotically (energy levels distributed according to Wigner-Dyson statistics) gives $\tau \propto L$ [see discussion below Eq. (11) and Ref. [9]]. This should be contrasted with standard bulk disorder which leads to a finite relaxation time independent of the system size and, for sufficiently large systems, localization.

V. CONCLUDING REMARKS

In this work we have presented a study of the time decay of correlation functions in quantum billiards with bulk or surface disorder. Our results indicate that at least for small values of the disorder parameter, the Lyapunov exponent is approximately given by the imaginary part of the self-energy induced by disorder. The results were obtained in the regime where no localization occurs and the systems show signs of chaotic behavior (eigenvalues distributed according to the Wigner-Dyson distribution). This is achieved by keeping constant l/L, when either L or the disorder parameter W are varied. Under these conditions, the Lyapunov exponent is proportional to $1/L$, and thus vanishes in the large L limit. We have also shown that although our finding somehow

agrees with Laughlin's guess concerning the existence of a relaxation time induced by chaos, important differences between the two approaches do exist (see above).

An interesting by-product of our study is that our numerical and analytical results indicate that a model with an amount of bulk impurities proportional to the linear size of the system L, behaves similarly to the model which only includes surface disorder proposed in [6]. Describing chaotic billiards by means of this type of bulk disorder (scaled disorder as referred to in [5]) could make easier the analysis of these systems by means of the supersymmetric sigma model [7,19].

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